Attendance Problems. **There are 5 blue, 4 red, 1 yellow and 2 green beads in a bag. Find the probability that a bead chosen at random from the bag is:**

1. blue **2.** green **3.** blue or red

4. blue or yellow **5.** not red **6.** not yellow

- I can determine whether events are independent or dependent.
- I can find the probability of independent and dependent events.

Vocabulary		
independent events	dependent events	conditional probability

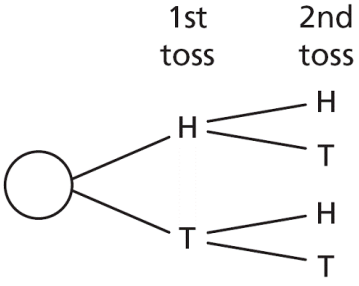
Common Core

CC.9-12.S.CP.3 Understand ... independence of *A* and *B* as ... the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.

CC.9-12.S.CP.6 Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*,...

Events are **independent events** if the occurrence of one event does not affect the probability of the other.

If a coin is tossed twice, its landing heads up on the first toss and landing heads up on the second toss are independent events. The outcome of one toss does not affect the probability of heads on the other toss. To find the probability of tossing heads twice, multiply the individual probabilities, $\frac{1}{2} \bullet \frac{1}{2}$ *or* $\frac{1}{4}$.



Probability of Independent Events

If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

Video Example 1.

A. What is the probability of spinning a 1 and then 1 again.

B. Find the probability of spinning red, then green and then 5 on the spinner.

1 Finding the Probability of Independent Events

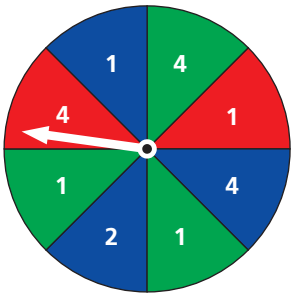
Find each probability.

A **spinning 4 and then 4 again on the spinner**

Spinning a 4 once does not affect the probability of spinning a 4 again, so the events are independent.

$P(4 \text{ and then } 4) = P(4) \cdot P(4)$

$\frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64}$ *3 of the 8 equal sectors are labeled 4.*



B **spinning red, then green, and then red on the spinner**

The result of any spin does not affect the probability of any other outcome.

$P(\text{red, then green, and then red}) = P(\text{red}) \cdot P(\text{green}) \cdot P(\text{red})$

$= \frac{1}{4} \cdot \frac{3}{8} \cdot \frac{1}{4} = \frac{3}{128}$ *2 of the 8 equal sectors are red; 3 are green.*

Example 1. A six-sided cube is labeled with the numbers 1, 2, 2, 3, 3, and 3. Four sides are colored red, one side is white, and one side is yellow. Find the probability.

A. Tossing 2 and then 2

B. Tossing red, then white, then yellow.

Guided Practice. Find each probability.

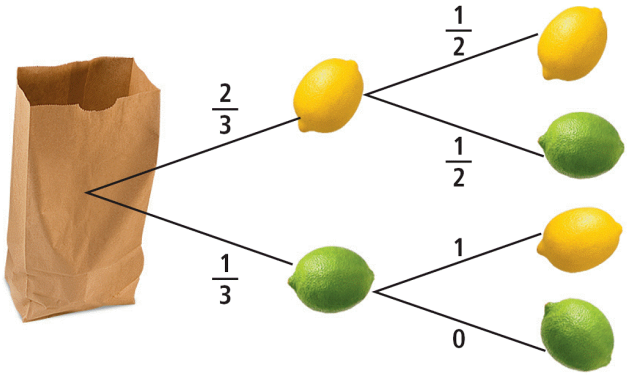
7. rolling a 6 on one number cube and a 6 on another number cube.

8. tossing heads, then heads, and then tails when tossing a coin 3 times

13-3 Assignment *(pp 891)* 10-11.

Events are **dependent events** if the occurrence of one event affects the probability of the other. For example, suppose that there are 2 lemons and 1 lime in a bag. If you pull out two pieces of fruit, the probabilities change depending on the outcome of the first.

The tree diagram shows the probabilities for choosing two pieces of fruit from a bag containing 2 lemons and 1 lime.



The probability of a specific event can be found by multiplying the probabilities on the branches that make up the event. For example, the probability of drawing two lemons is $\frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$.

To find the probability of dependent events, you can use **conditional probability** $P(B|A)$, the probability of event B, given that event A has occurred.

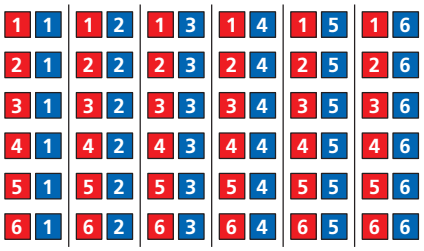
Probability of Dependent Events

If A and B are dependent events, then $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$, where $P(B \mid A)$ is the probability of B, given that A has occurred.

Video Example 2. Two number cubes are rolled-one red and one white. Explain why the events are dependent. Find the probability. The sum is greater than 9 and the red cube shows a five.

EXAMPLE 2 Finding the Probability of Dependent Events

Two number cubes are rolled—one red and one blue. Explain why the events are dependent. Then find the indicated probability.



A **The red cube shows a 1, and the sum is less than 4.**

Step 1 Explain why the events are dependent.

$P(\text{red } 1) = \frac{6}{36} = \frac{1}{6}$ *Of 36 outcomes, 6 have a red 1.*

$P(\text{sum} < 4 \mid \text{red } 1) = \frac{2}{6} = \frac{1}{3}$ *Of 6 outcomes with a red 1, 2 have a sum less than 4.*

The events "the red cube shows a 1" and "the sum is less than 4" are dependent because $P(\text{sum} < 4)$ is different when it is known that a red 1 has occurred.

Step 2 Find the probability.

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$
$$P(\text{red } 1 \text{ and sum} < 4) = P(\text{red } 1) \cdot P(\text{sum} < 4 \mid \text{red } 1)$$
$$= \frac{1}{6} \cdot \frac{2}{3} = \frac{1}{18}$$

Helpful Hint

In Example 2A, you can check to see that 2 of the 36 outcomes, or $\frac{1}{18}$, have a red 1 and a sum less than 4: (1, 1) and (1, 2).

Explain why the events are dependent. Then find the indicated probability.

B **The blue cube shows a multiple of 3, and the sum is 8.**

The events are dependent because $P(\text{sum is } 8)$ is different when the blue cube shows a multiple of 3.

$P(\text{blue multiple of } 3) = \frac{2}{6} = \frac{1}{3}$ *Of 6 outcomes for blue, 2 have a multiple of 3.*

$P(\text{sum is } 8 \mid \text{blue multiple of } 3) = \frac{2}{12} = \frac{1}{6}$ *Of 12 outcomes that have a blue multiple of 3, 2 have a sum 8.*

$P(\text{blue multiple of } 3 \text{ and sum is } 8) =$

$P(\text{blue multiple of } 3) \cdot P(\text{sum is } 8 \mid \text{blue multiple of } 3) = \left(\frac{1}{3}\right)\left(\frac{1}{6}\right) = \frac{1}{18}$

Example 2. Two number cubes are rolled–one white and one yellow. Explain why the events are dependent. Then find the indicated probability.

A. The white cube shows a 6 and the sum is greater than 9 .

B. The yellow cube shows an even number and the sum is 5.

9. Guided Practice. Two number cubes are rolled—one red and one black. Explain why the events are dependent, and then find the indicated probability.
The red cube shows a number greater than 4, and the sum is greater than 9.

13-3 Assignment *(pp 891-893)* 10-**12**

Conditional probability often applies when data fall into categories.

<u>Video Example 3</u>. The table shows the approximate distribution of votes in Texas' five largest counties in the 2004 presidential election. Suppose that one voter is selected at random. Find each probability.

Largest Texas Counties' Votes for President 2004 (thousands)			
County	Bush	Kerry	Other
Harris	581	472	5
Dallas	345	336	4
Tarrant	349	207	3
Bexar	260	210	3
Travis	148	197	5

A. A voter from Travis County voted for Kerry.

B. A voter was from Harris County voted for Bush.

3 Using a Table to Find Conditional Probability

Largest Texas Counties' Votes for President 2004 (thousands)			
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The table shows the approximate distribution of votes in Texas' five largest counties in the 2004 presidential election. Find each probability.

A **that a voter from Tarrant County voted for George Bush**

$$P(\text{Bush} \mid \text{Tarrant}) = \frac{349}{559} \approx 0.624$$

Use the Tarrant row. Of 559,000 Tarrant voters, 349,000 voted for Bush.

B **that a voter voted for John Kerry and was from Dallas County**

$$P(\text{Dallas} \mid \text{Kerry}) = \frac{336}{1422}$$

Of 1,422,000 who voted for Kerry, 336,000 were from Dallas County.

$$P(\text{Kerry and Dallas} \mid \text{Kerry}) = \frac{1422}{3125} \cdot \frac{336}{1422} \approx 0.108$$

There were 3,125,000 total voters.

<u>Example 3</u>. The table shows domestic migration from 1995 to 2000. A person is randomly selected. Find each probability.

Domestic Migration by Region (thousands)		
Region	**Immigrants**	**Emigrants**
Northeast	1537	2808
Midwest	2410	2951
South	5042	3243
West	2666	2654

A. that an emigrant is from the West

B. that someone selected from the South region is an immigrant

C. that someone selected is an emigrant and is from the Midwest

<u>Guided Practice</u>.

10. that a voter from Travis county voted for someone other than George Bush or John Kerry.

11. that a voter was from Harris county and voted for George Bush.

13-3 Assignment *(pp 891-892)* 10-12, **15-16**

In many cases involving random selection, events are independent when there is replacement and dependent when there is not replacement.

Remember!

A standard card deck contains 4 suits of 13 cards each. The face cards are the jacks, queens, and kings.

Video Example 4. Two cards are drawn from a deck of 52. Determine whether the events are independent or dependent. Find the probability.

A. Selecting two red cards when the first one is replaced.

B. Selecting a queen or jack and then an ace without replacement.

4 Determining Whether Events Are Independent or Dependent

Two cards are drawn from a deck of 52. Determine whether the events are independent or dependent. Find the probability.

A **selecting two aces when the first card is replaced**

Replacing the first card means that the occurrence of the first selection will not affect the probability of the second selection, so the events are independent.

$$P(\text{ace} \mid \text{ace on first draw}) = P(\text{ace}) \cdot P(\text{ace})$$

$$= \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$$

4 of the 52 cards are aces.

B **selecting a face card and then a 7 when the first card is not replaced**

Not replacing the first card means that there will be fewer cards to choose from, affecting the probability of the second selection, so the events are dependent.

$$P(\text{face card}) \cdot P(7 \mid \text{first card was a face card})$$

$$= \frac{12}{52} \cdot \frac{4}{51} = \frac{4}{221}$$

There are 12 face cards, four 7's and 51 cards available for the second selection.

<u>Example 4</u>. Two cards are drawn from a deck of 52. Determine whether the events are independent or dependent. Find the probability.

A. selecting two hearts when the first card is replaced

B. selecting two hearts when the first card is not replaced

C. a queen is drawn, is not replaced, and then a king is drawn.

Guided Practice. A bag contains 10 beads—2 black, 3 white, and 5 red. A bead is selected at random. Determine whether the events are independent or dependent. Find the indicated probability.

12. selecting a white bead, replacing it, and then selecting a red bead.

13. selecting a white bead, not replacing it, and then selecting a red bead

14. selecting 3 nonred beads without replacement

13-3 Assignment

- *(pp 891-893)* 10-12, 15-18, 22, 24, 25, 28, 29, 31-34.
- Ready to Go On Lesson 13A